A SIMPLE 9/7-TAP WAVELET FILTER BASED ON LIFTING SCHEME

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ABSTRACT

Based on lifting scheme and the construction theorem of biorthogonal wavelet, we propose a new symmetric biorthogonal 9/7-tap wavelet called LS97. Compared to Cohen-Daubechies-Feauveau 9/7-tap (CDF 9/7-tap) wavelet adopted by JPEG2000, when new wavelet is applied to image coding, the compression performance is exactly the same as that of CDF 9/7-tap wavelet, while computational complexity is reduced remarkably.

1. INTRODUCTION

In the new still picture-compression standard JPEG2000 [1], biorthogonal CDF 9/7-tap wavelet developed by Cohen-Daubechies-Feauveau [2] was recommended as standard transform coding. To reduce the complexity, Daubechies and Sweldens [3] proposed a lifting scheme for implementing CDF 9/7-tap wavelet. Since the coefficients of the CDF 9/7-tap wavelet are all irrational numbers, [4] suggested approximating the coefficients of the CDF 9/7-tap wavelet by using binary fraction, in which 26 integer additions and 18 shifts are needed per two wavelet coefficients. However, the compression performance of binary 9/7-tap wavelet [4] is inferior to that of CDF 9/7-tap wavelet. The purpose of this Letter is to develop a new biorthogonal 9/7-tap wavelet with a simple binary fraction, in which the compression performance is nearly the same as that of CDF 9/7-tap wavelet while computational complexity is reduced considerably. The simulation shows that the new biorthogonal 9/7-tap wavelet can be considered as a very good alternative to CDF 9/7-tap wavelet.

2. NEW BIORTHOGONAL 9/7-TAP WAVELET

Let the functions of analysis low-pass symmetric filter and synthesis low-pass symmetric filter be $H(\omega)=h_0+2\sum_{n=1}^{L_1} h_n \cos n\omega$ and $G(\omega)=g_0+2\sum_{n=1}^{L_2} g_n \cos n\omega$ respectively. To construct biorthogonal wavelet, Cohen, Daubechies and Feauveau [2] developed the following construction theorem.

Theorem([2]). Let $H(\omega)=\sqrt{2}(\frac{1+e^{-i\omega}}{2})^N P(\omega)$ and $G(\omega)=\sqrt{2}(\frac{1+e^{-i\omega}}{2})^{\tilde{N}} \tilde{P}(\omega)$, where $P(\omega)$ and $\tilde{P}(\omega)$ are polynomials about $e^{-i\omega}$, then $H(\omega)$ and $G(\omega)$ construct biorthogonal wavelet, if the following conditions are true:

(i) Normalization condition: $H(0)=\sqrt{2}$, $G(0)=\sqrt{2}$;

(ii) Inequalities constraints:

$$\sup_{\omega\in[0,2\pi)} |P(\omega)|<2^{N-1},\ \sup_{\omega\in[0,2\pi)} |\tilde{P}(\omega)|<2^{\tilde{N}-1};$$

(iii) Perfect reconstruction condition:

$$H(\omega)\overline{G(\omega)}+H(\omega+\pi)\overline{G(\omega+\pi)}=2.$$

In practice especially in image processing applications, to reduce the complexity of wavelet transform, the FIR filters should be within limited lengths. For example, in the design of the CDF 9/7-tap wavelet [2], the lengths of FIR filters are taken by $L_1=4$ and $L_2=3$ respectively, and another group of constants N and $\tilde{N}$ are taken by $N=\tilde{N}=4$. In the design of new 9/7-tap biorthogonal wavelet, we change the value of N and $\tilde{N}$, and take $N=2$, $\tilde{N}=4$, in this case, we have

$$H(\omega)=\sqrt{2}(\frac{1+e^{-i\omega}}{2})^2 P(\omega),$$

$$G(\omega)=\sqrt{2}(\frac{1+e^{-i\omega}}{2})^4 \tilde{P}(\omega) \quad (1)$$

Based on the construction Theorem, in the following we consider solving for $H(\omega)$ and $G(\omega)$. First, from normalization condition we get

$$h_0+2\sum_{n=1}^{4} h_n=\sqrt{2},\ \ g_0+2\sum_{n=1}^{3} g_n=\sqrt{2}. \quad (2)$$

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Using (1) results in $\frac{d^k}{d\omega^k} H|_{\omega=\pi} = 0,\ k = 0, 1$ and $\frac{d^k}{d\omega^k} G|_{\omega=\pi} = 0,\ k = 0, 1, 2, 3$. Since $H(\omega)$ and $G(\omega)$ are functions about $\cos(\omega)$, we have $\frac{d^{2m+1}}{d\omega^{2m+1}} H|_{\omega=\pi} \equiv 0$, $\frac{d^{2m+1}}{d\omega^{2m+1}} G|_{\omega=\pi} \equiv 0$ for $m \ge 0$. So the above equations can be further simplified into

$$h_0 + 2\sum_{n=1}^{4}(-1)^n h_n = 0,\ g_0 + 2\sum_{n=1}^{3}(-1)^n g_n = 0,$$
$$2\sum_{n=1}^{3} n^2(-1)^n g_n = 0 \tag{3}$$

We now combine (3), (4) and lifting scheme to determine new 9/7-tap biorthogonal wavelet. In [3] the analysis polyphase matrix $P_a(z)$ of the 9/7-tap wavelet was factored as

$$P_a(z) = \begin{bmatrix} \zeta & 0 \\ 0 & -1/\zeta \end{bmatrix} \begin{bmatrix} 1 & \delta(1+z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \gamma(1+z^{-1}) & 1 \end{bmatrix} \begin{bmatrix} 1 & \beta(1+z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \alpha(1+z^{-1}) & 1 \end{bmatrix} \tag{4}$$

Using symmetric property of the coefficients of h and g, $P_a(z)$ can be equivalently expressed by

$$P_a(z) = \begin{bmatrix} h_0 + h_2(z+z^{-1}) + h_4(z^2+z^{-2}) & g_1(1+z^{-1}) + g_3(z+z^{-2}) \\ h_1(z+1) + h_3(z^2+z^{-1}) & -g_0 - g_2(z+z^{-1}) \end{bmatrix} \tag{5}$$

Comparing (4) and (5) we have

$$\begin{cases} h_0 = (1+2\alpha\beta+2\alpha\delta+2\gamma\delta+6\alpha\beta\gamma\delta)\zeta \\ h_1 = (3\beta\gamma\delta+\beta+\delta)\zeta \\ h_2 = (\alpha\beta+\alpha\delta+\gamma\delta+4\alpha\beta\gamma\delta)\zeta \\ h_3 = \beta\gamma\delta\zeta \\ h_4 = \alpha\beta\gamma\delta\zeta \\ g_0 = (2\beta\gamma+1)/\zeta \\ g_1 = -(3\alpha\beta\gamma+\alpha+\gamma)/\zeta \\ g_2 = \beta\gamma/\zeta \\ g_3 = -\alpha\beta\gamma/\zeta \end{cases} \tag{6}$$

Combining (6), (2) and (3) yields

$$\begin{cases} \beta = -\frac{1}{4(1+2\alpha)^2} \\ \gamma = \frac{-1-4\alpha-4\alpha^2}{1+4\alpha} \\ \delta = \frac{1}{16}\left(4 - \frac{2+4\alpha}{(1+2\alpha)^4} + \frac{1-8\alpha}{(1+2\alpha)^2}\right) \\ \zeta = \frac{2\sqrt{2}(1+2\alpha)}{1+4\alpha} \end{cases} \tag{7}$$

(7) shows that β, γ, δ and ζ can be all expressed by a free parameter α. In fact, let $\alpha = -1.5861343420\cdots$ (an irrational number), then the CDF 9/7-tap wavelet is achieved, in this case, we get

$$\begin{cases} \alpha = -1.5861343420\cdots \\ \beta = -0.0529801185\cdots \\ \gamma = 0.8828110755\cdots \\ \delta = 0.4435068520\cdots \\ \zeta = 1.1496043988\cdots \end{cases}$$

where parameters are all irrational numbers. To simplify computation we take a simple parameter $\alpha = -\frac{3}{2}$, in this time, we get

$$\begin{cases} \alpha = -3/2 \\ \beta = -1/16 \\ \gamma = 4/5 \\ \delta = 15/32 \\ \zeta = 4\sqrt{2}/5 \end{cases}$$

and the corresponding filters coefficients has the form

$$\begin{array}{ll} h_0 = 19\sqrt{2}/32 & g_0 = 9\sqrt{2}/16 \\ h_1 = 43\sqrt{2}/160 & g_1 = 19\sqrt{2}/64 \\ h_2 = -3\sqrt{2}/40 & g_2 = -\sqrt{2}/32 \\ h_3 = -3\sqrt{2}/160 & g_3 = -3\sqrt{2}/64 \\ h_4 = 9\sqrt{2}/320 & \end{array}$$

We call the filters as LS97 filters.

Furthermore, we can prove the new LS97 filters construct biorthogonal wavelet. In fact, we have

$$P(\omega) = \left(1 + \frac{3}{8}\cos\omega - \frac{3}{5}\cos 2\omega + \frac{9}{40}\cos 3\omega\right)e^{i\omega},$$

$$\tilde{P}(\omega) = \left(\frac{5}{2} - \frac{3}{2}\cos\omega\right)e^{2i\omega}.$$ Therefore,

$\sup_{\omega\in[0,2\pi)} |P(\omega)| = \frac{8}{5} + \frac{7}{405} < 2^1$, $\sup_{\omega\in[0,2\pi)} |\tilde{P}(\omega)| = 4 < 2^3$, from the construction Theorem we conclude that LS97 filters construct biorthogonal wavelet.

3. EXPERIMENTAL RESULTS

Obviously, compared with CDF 9/7-tap wavelet, the coefficients of LS97 wavelet are greatly simplified. For example, for LS97 wavelet only two floating-point and 16 integer operations are needed per two wavelet coefficients, while for CDF 9/7-tap wavelet, 8 floating–point additions and 6 floating-point multiplications must be used. In the following we focus on the comparison of compression performance. In the simulation, we try the following four configures:

i. CDF-CDF: CDF 9/7-tap wavelet is used in both encoder and decoder;
ii. LS-LS: LS97 wavelet is used in both encoder and decoder;
iii. CDF-LS: CDF 9/7-tap wavelet is used in encoder, while LS97 wavelet in decoder;
iv. LS-CDF: LS97 wavelet is used in encoder, while CDF 9/7-tap wavelet in decoder;

To be fair, the same EBCOT algorithm [5] adopted by JPEG2000 is utilized to encode the coefficients of every configure. The objective coding results (PSNR in dB) for standard 512×512 Lena,Goldhill and Barbara test images are tabulated in Table 1~3. It is easily to see that the performance of LS-LS is exactly the same as that of CDF-CDF. Furthermore, CDF-LS and LS-CDF are also close to CDF-CDF when the compression ratio is greater than 16. The visual quality of their reconstructed images is also exactly the same, as demonstrated in Fig. 1.

C.R.	CDF-CDF	LS-LS	CDF-LS	LS-CDF
1:8	40.38	40.38	39.93	39.96
1:16	37.29	37.29	37.04	37.10
1:32	34.16	34.16	34.02	34.09
1:64	31.01	31.02	30.93	31.00
1:128	28.15	28.17	28.10	28.17

Table 1: PSNR evaluation for Lena, in dB

C.R.	CDF-CDF	LS-LS	CDF-LS	LS-CDF
1:8	36.60	36.61	36.46	36.53
1:16	33.25	33.27	33.17	33.24
1:32	30.54	30.54	30.48	30.54
1:64	28.48	28.50	28.44	28.50
1:128	26.59	26.62	26.56	26.63

Table 2: PSNR evaluation for Goldhill, in dB

C.R.	CDF-CDF	LS-LS	CDF-LS	LS-CDF
1:8	37.19	37.21	36.90	36.95
1:16	32.31	32.32	32.21	32.23
1:32	28.37	28.46	28.33	28.42
1:64	25.45	25.46	25.43	25.44
1:128	23.38	23.36	23.36	23.36

Table 3: PSNR evaluation for Barbara, in dB

4. CONCLUSIONS

We have presented a new biorthogonal 9/7-tap wavelet with simple coefficients, so computational complexity is reduced greatly compared to the well-known CDF 9/7-tap wavelet. The simulation shows new 9/7-tap wavelet is a very ideal alternative to CDF 9/7-tap wavelet.

5. REFERENCES

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(a) CDF-CDF

(c) CDF-LS

(b) LS-LS

(d) LS-CDF

Fig.1 Quarter portions of reconstructed Barbara images at 1:32 compression ration